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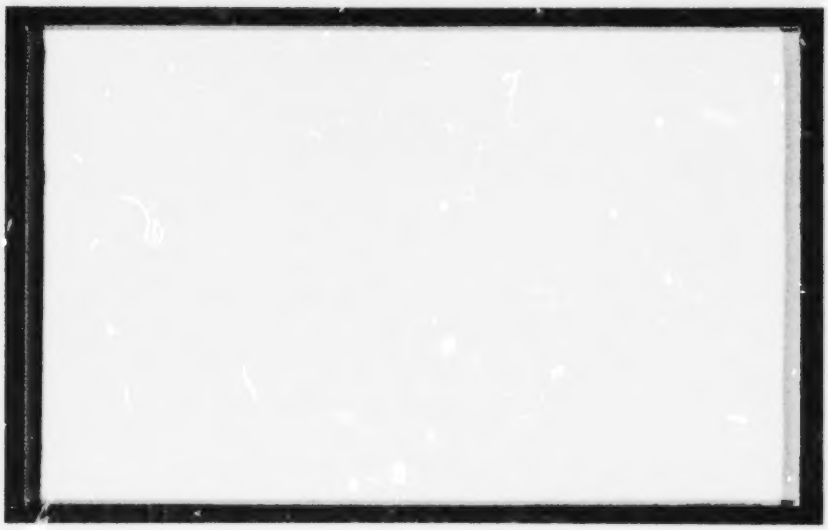

CENTRE DE RECHERCHE SUR LES TRANSPORTS

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TRANSCOL
A MULTI-PERIOD SCHOOL BUS ROUTING AND SCHEDULING SYSTEM

by

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ABSTRACT

This paper describes a modular system called TRANSCOL for planning and organizing transportation in large regional school system with approximately 60 schools and 20,000 students. In urban areas, students are picked up (and dropped off) at stops which are either specified by the authorities or selected by TRANSCOL. In rural areas, students are picked up (and dropped off) at their homes. Those going to high schools located in urban areas may use connection points to take more than one bus. The originality of TRANSCOL lies in the fact that it builds the school time-tables within some range specified by the authorities so as to reduce the number of buses required. Finally, the system schedules the routes in time. Numerical results are reported and large savings are identified

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RÉSUMÉ

TRANSCOL est un système modulaire pour la planification et l'organisation du transport d'une commission scolaire. En milieu urbain les étudiants sont pris et retournés à des arrêts qui peuvent être spécifiés par les responsables ou sélectionnés par le système TRANSCOL lui-même. En milieu rural, par contre, les étudiants sont pris et retournés à leur domicile. Ceux qui fréquentent des écoles secondaires situées en milieu urbain peuvent avoir à changer d'autobus à certains points de correspondance. L'originalité de TRANSCOL vient du fait que le système permet d'établir les horaires de chaque école à l'intérieur d'une fenêtre de temps spécifiée par les responsables. Le but poursuivi ainsi est la réduction de la flotte d'autobus requise. En dernier lieu, les parcours sont cédulés pour favoriser le chaînage de ces derniers. Des résultats numériques sont présentés, indiquant que le système permet de réaliser des économies appréciables.

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1. INTRODUCTION

This paper describes a modular system called TRANSCOL for planning and organizing transportation in a large regional school system. The originality of TRANSCOL lies in the fact that it builds the school time-tables within some ranges specified by the authorities so as to reduce the number of buses required.

In the Québec school system, the transportation of students by bus is required because of low population density and because the Government has decided to establish large regional high schools. Each regional school board is responsible for a large territory including approximately 60 elementary and high schools and 20,000 students. Thus transportation planning for a regional school board gives rise to very large mathematical models (see [25] for example). Because these large models are difficult to solve, a more modular approach is proposed in this paper. Each of these modules considers a component of the problem and generates input to other modules. TRANSCOL, the system proposed, includes four modules that are executed sequentially.

a) The LOCATION module specifies the data for the other modules.

b) The BUS ROUTE GENERATOR module constructs productive journey. Each route is carried out by a bus and is characterized by a place of origin, a destination, a duration and a cost. Two different generators are used. In urban areas, students are picked up (and dropped off) at stops. Because of low population density in rural areas, students are picked up (and dropped off) at their homes. These two problems are quite different since the first one is a variant of the travelling salesman problem while the second is a variant of the chinese postman problem.

c) The SCHOOL SCHEDULING module specifies the starting and ending time for each school with the objective of maximizing the number of routes that can be executed sequentially by the same bus.

d) The ROUTE SCHEDULING module determines schedules for all the routes so as to minimize the number of buses and travel costs while respecting network and time constraints.

It might appear that it is sufficient to schedule the bus in the morning and to reverse the circuits in the afternoon, or that the morning problem and the afternoon problem are independent. This is not the case since different grades have school-days of differing lengths, and a decision which might improve the situation in the morning could be disastrous in the afternoon, and vice versa. Thus, morning and afternoon scheduling has to be done simultaneously, even if the afternoon routes are the reversed morning routes (as in our tests).

Before we present each module, we specify the constraints:

i) For each school an earliest starting time (in the morning) and a latest ending time (in the afternoon) are specified for its routes.

ii) For each school, an interval of time (from 8:30 to 9:10, for example) for its starting time in the morning, and an interval of time length (from 60 min. to 90 min. for example) for the lunch break are specified. Hence different choices of starting time and lunch time length specify different school ending times because the in-school time is predetermined.

iii) There is an upper bound on the waiting time for students between their arrival at school and the school starting time and between the

school ending time and the bus' departure: 30 minutes is the value used in our tests.

iv) The students must not arrive later than 10 minutes before the school starting time and must not depart earlier than 10 minutes after the school ending time.

v) There is an upper bound on the transportation time for a student.

vi) There is an upper bound on the walking distance for a student.

vii) There are upper bounds on the number of students on routes (i.e. bus capacities).

There are two cost components: the operating cost for transportation and the rental cost of the buses. Since the first component is quite stable whatever strategy is used, the objective is to minimize the rental cost. This is achieved by scheduling the school starting and ending times and the routes so as to use the same bus on as many routes as possible.

This routing and scheduling problem has been partly examined previously. Angel *et al.* [1], Bennet and Gazes [2] and Dulac *et al.* [16] deal with the routing problem. Bodin and Berman [3] and Newton and Thomas [27] study the routing and the route scheduling problems. The approach in [3] is to partition the time horizon, and to link the routes in adjacent time periods by solving a transportation problem. Our approach is similar, but the time horizon is not partitioned a priori. Eloff [28] solves successive matching problems while Swersey and Ballard [30] solve one linear problem where the time window associated with the starting time of a route is discretized. Desrosiers et al. [13,14] propose an exact solution

procedure based on decomposition and branch-and-bound. The school scheduling aspect in TRANSCOL is original.

Having specified the problem, we now review the first two modules (location and bus route generator) in Section 2. The school scheduling and route scheduling modules are described in Sections 3 and 4, respectively. Section 5 includes some numerical results while Section 6 offers some concluding remarks.

2. GENERATING ROUTES

In the location module, the road network is specified. Each student's address and school are given. Finally, the locations of the schools and connection points (to be discussed later) are specified. These data are assumed to be available. The purpose of this module is to prepare the data for the other modules.

In urban areas it is assumed (by the school board) that the students walk from their homes to the stops (and back). A set of potential stops has to be specified by the school board, otherwise all street intersections are potential stops.

Since in rural areas the students are picked up at and returned to their homes, two different generators are used.

In urban areas [6]

In general, students are located on edges (street segments), but since it is easier to compute distances between vertices of a network,

all the students on an edge are first assigned to the incident vertex (referred to as mini-stop by Bodin and Berman [3] which is closest to a potential stop.

Several versions of well known vehicle routing techniques such as Clarke and Wright [8], Insertion [29] and Newton and Thomas [27] have been analysed [16,19]. The purpose of these techniques is to minimize the total length of the routes, despite the fact that in urban areas the most binding constraint is the upper bound on the number of students on each route (corresponding to the bus capacity). The upper bound on the length of each route (transportation time) is irrelevant because of the high population density of the area. The objective is then to generate a minimum number of routes, filling them as close as possible to their capacity. As an alternative, we developed a clustering approach based on simple districting algorithms. The results in [6] indicate improvement both in configuration and in the number of routes compared to the previously tested techniques. This approach uses the "group first route second" routing strategy : as such it is in the same category as the sweep method proposed by Gillet and Miller [21] and it can be seen as a generalization of this method.

To determine clusters, the algorithm divides the set of mini-stops into a minimum number of compact groups, taking into account the route capacity limit. This is achieved through exchanges of mini-stops in order to minimize the sum of the square of the distances between each mini-stop and the centroid of its cluster. This sum is defined as the compacity

measure. Then for each cluster, a route and the stops along this route are determined, using a 2-OPT procedure [8,9]. Once the routes are specified, further exchange procedures are used to guarantee that each student is assigned to the nearest stop, to satisfy the capacity constraint on each route and to reduce total length.

In rural areas [7]

In rural areas, routes along edges must be generated. This problem is referred to as the capacitated chinese postman problem [24]. Students are directed either to school or to a "connection point". Indeed, since most of the high schools are located in cities, many students from rural areas are first picked up at their homes and transported to certain gathering points. They are then taken to their respective schools via express routes. Routes are assigned to connection points in the step following route generation.

In rural areas, the upper bound on the transportation time for the students is the most relevant constraint. A parallel insertion technique generating several routes simultaneously is used to produce a more balanced set of routes. During the first step of the parallel insertion, the remaining farthest edge from the destination is inserted in the "best" route. If it is not possible to find an insertion according to the constraints, a new route is started with this edge. During the second step, the remaining edges are inserted into existing routes according to different strategies. It may be necessary to increase the number of routes to find a feasible solution. Each time an edge is introduced into a route,

a 2-OPT procedure is called. Exchange procedures take place at every step of the heuristic in order to reduce total length and to avoid an increase in the number of routes. This approach is fully described in [7].

3. SCHOOL SCHEDULING [4,10]

The purpose of this module is to schedule the school starting and ending times in such a way that the same bus can be used on as many routes as possible. Of course, if two routes are executed by the same bus, there must be enough time between the beginning of the second and the end of the first to allow the bus to go from the end vertex of the first route to the first vertex of the second route. In the initial approach, this constraint is relaxed and a lower bound on the number of buses required is obtained. In the second approach, this constraint is included and an improved lower bound is generated.

The time scale is divided into periods of ten minutes and an integer number of periods is accodiated with each route. For instance, if a route requires 25 minutes to be executed, then three periods are associated with it. Constraints iii and iv (Section 1) require that each route be completed at most three periods and at least one period before the school starting time. Similar constraints exist during the afternoon relative to the school ending time. Thus, in general, most of the routes can be completed at three different moments in the morning and initialized at three different moments in the afternoon. Since each route has to be executed in the morning and in the afternoon, this generates nine possible run combinations for a route. Note that for the longer routes the number

of possible run combinations is reduced in order to reduce the overall inconvenience to the students using these routes. In figure 1, the nine possible run combinations of a route requiring two periods are illustrated. A "1" in a time period indicates that the route is executed during this time period.

3.1 Initial approach

The objective is to minimize the number of run combinations taking place during the (critical) period where the maximal (over all periods) number of these takes place. This problem can be formulated as a Min Max 0-1 program as follows :

Min z

$$\text{Subject to} \quad \sum_{s=1}^{n} \sum_{j=1}^{J(s)} \sum_{i=1}^{I(j)} a(k,\ell,s,i,j)\, x(\ell,s,i,j) \le z, \quad 1 \le k \le m \qquad (1)$$

$$\sum_{i=1}^{I(j)} x(\ell,s,i,j,) = y(\ell,s) \qquad 1 \le j \le J(s), \quad 1 \le \ell \le L(s), \quad 1 \le s \le n \ , \qquad (2)$$

$$\sum_{\ell=1}^{L(s)} y(\ell,s) = 1 \ , \qquad 1 \le s \le n \ , \qquad (3)$$

$x(\ell,s,i,j) = 0$ or 1, $1 \le i \le I(j)$, $1 \le j \le J(s)$, $1 \le \ell \le L(s)$, $1 \le s \le n$,

$y(\ell,s) = 0$ or 1, $1 \le \ell \le L(s)$, $1 \le s \le n$,

Time	Run combination								
	1	2	3	4	5	6	7	8	9
7:50									
5th period before						1		1	1
8:00									
4th period before			1	1		1	1	1	1
8:10									
3rd period before	1	1	1	1	1		1		
8:20									
2nd period before	1	1			1				
8:30									
1st period before									
8:40 School starting time									
.									
.									
.									
3:10 School ending time									
1st period after									
3:20									
2nd period after	1		1			1			
3:30									
3rd period after	1	1	1	1		1		1	
3:40									
4th period after		1		1	1		1	1	1
3:50									
5th period after					1		1		1
4:00									

Figure 1

Example of run combinations

where n : number of schools,

m : number of periods,

L(s): number of possible schedules for school s,

J(s): number of routes for school s,

I(j): number of possible run combinations for route j,

$$a(k,\ell,s,i,j,) = \begin{cases} 1 \text{ if the ith possible run combination for route j in the } \ell\text{th possible schedule for school s is executed during period k} \\ 0 \text{ otherwise.} \end{cases}$$

The decisions variables $x(s,\ell,i,j)$ and $y(\ell,s)$ are specified as follows

$$x(\ell,s,i,j) = \begin{cases} 1 \text{ if the ith possible run combination for route j in the } \ell\text{th possible schedule for school s is selected} \\ 0 \text{ otherwise} \end{cases}$$

$$y(\ell,s) = \begin{cases} 1 \text{ if the } \ell\text{th schedule for school s is selected} \\ 0 \text{ otherwise.} \end{cases}$$

Constraints (3) indicate that one schedule has to be selected for each school, while constraints (2) indicate that one possible run combination for each route is chosen for the selected schedule. The left hand side of (1) evaluates the number of run combinations taking place during period k.

The main difficulty is the size, and hence we adopt a decomposition approach.

For a fixed schedule ℓ for a school s the following subproblem can be formulated :

$$\text{Min } z$$

$$\text{Subject to } \sum_{j=1}^{J(s)} \sum_{i=1}^{I(j)} a(k,\ell,s,i,j)\, x(\ell,s,i,j) \le z \quad , \qquad 1 \le k \le m$$

$$\sum_{i=1}^{I(j)} x(\ell,s,i,j) = 1, \qquad 1 \le j \le J(s), \qquad (P_{\ell s})$$

$$x(\ell,s,i,j) = 0 \text{ or } 1 \qquad 1 \le i \le I(j),\ 1 \le j \le J(s).$$

Such a problem is illustrated if Figure 2 for a schedule with starting time 08:40 and ending time 15:40. Let

$$u_k(\ell,s) = \sum_{j=1}^{J(s)} \sum_{i=1}^{I(j)} a(k,\ell,s,i,j)\, x(\ell,s,i,j) \qquad 1 \le k \le m$$

and let $u(\ell,s) = [u_k(\ell,s)] \in E^m$ be the <u>utilization vector</u> associated with a feasible solution for the subproblem $(P_{\ell s})$. For instance suppose that the run combinations 5 and 6 are selected for routes #1 and #2, respectively. Then the utilization vector is as shown in Figure 2.

Using this notation and given the utilization vector associated with a specified feasible solution for each pair ℓ,s, the following problem can be written

$$\text{Min } z$$

$$\text{Subject to } \sum_{s=1}^{n} \sum_{\ell=1}^{L(s)} u_k(\ell,s)\, y(\ell,s) \le z \quad , \qquad 1 \le k \le m$$

$$\sum_{\ell=1}^{L(s)} y(\ell,s) = 1 \ , \qquad 1 \le s \le n \qquad (P)$$

$$y(\ell,s) = 0 \text{ or } 1 \ , \qquad 1 \le \ell \le L(s),\ 1 \le s \le n \ .$$

Notice that this problem is a restriction [20] of the original one since for each pair ℓ,s one utilization vector among many is selected.

Time	Combined running for																Utilization vector
	Route #1									Route #2										
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	
7:50																			
5th period before						1		1	1						1		1	1	1
8:00																			
4th period before			1	1		1	1	1	1			1	1		1	1	1	1	1
8:10																			
3rd period before	1	1	1	1	1		1			1	1	1	1	1		1			1
8:20																			
2nd period before	1	1			1					1	1			1					1
8:30																			
1st period before																			0
8:40 School starting time																			0
																			...
3:10 School ending time																			0
1st period after																			0
3:20																			
2nd period after	1		1			1				1		1			1				1
3:30																			
3rd period after	1	1	1	1		1		1		1	1	1	1		1		1		1
3:40																			
4th period after		1		1	1		1	1	1		1		1	1		1	1	1	1
3:50																			
5th period after					1		1		1					1		1		1	1
4:00																			

Figure 2

Initial approach for a school

A problem with three schools is illustrated in Figure 3. With each feasible solution is associated a <u>global utilization vector</u> $U = [u_k]$

where $$u_k = \sum_{s=1}^{n} \sum_{\ell=1}^{L(s)} u_k(\ell,s)\, y(\ell,s).$$

For instance, if schedules 3,2 and 3 are selected for schools 1, 2 and 3 respectively, then the global utilization vector is exibited in Figure 3. Notice that the structures of (P) and ($P_{\ell s}$) are identical.

The heuristic approach to solve the problem can now be summarized. First, determine utilization vectors for the different schedules (each specified by a starting and an ending time) of each school. In practice, instead of solving ($P_{\ell s}$) for each s and ℓ, the initial utilization vector is specified by adopting the first run combination for each route. This arbitrary choice reduces the solution time and is justified by the fact that the choice of run combination is modified later on. Then we proceed to select a schedule for each school by solving (P).

At this stage, a schedule has been selected for each school. Then, the problem of selecting the run combinations of the routes is solved once more for each school given the schedules selected. The purpose is to modify the selection of the run combinations (i.e. to determine a new utilization vector for the selected schedule of the school) given the selected utilization vectors for the other schools. The structure of the problem for school $\bar{s}$ is similar to (P) and ($P_{\ell s}$):

Time	Utilization vectors for different schedules of																Global utilization vector
	School 1				School 2						School 3						
	1	2	3	4	1	2	3	4	5	6	1	2	3	4	5	6	
7:20																	
7:30					2	2					1	1	1	1			3
7:40					2	2	2	2			2	2	2	2	1	1	4
7:50	1	1			2	2	2	2	2	2	3	3	2	2	2	2	4
8:00	1	1	1	1	1	1	2	2	2	2	3	3	3	3	2	2	5
8:10	1	1	1	1	1	1	1	1	2	2	3	3	2	2	3	3	4
8:20	1	1	1	1			1	1	1	1			2	2	2	2	3
8:30			1	1					1	1					2	2	1
8:40																	0
⋮																	⋮
3:00																	
3:10																	0
3:20	1										1						0
3:30	1	1	1								2	1	1				2
3:40	1	1	1	1	1						2	2	2	1	1		3
3:50	1	1	1	1	1	1	1				3	2	2	2	2	1	4
4:00		1	1	1	2	1	1	1	1		2	3	3	2	2	2	5
4:10				1	2	2	2	1	1	1	2	2	2	3	3	2	4
4:20					2	2	2	2	2	1		2	2	2	2	3	4
4:30						2	2	2	2	2				2	2	2	2
4:40								2	2	2						2	0
4:50										2							0

Figure 3

School scheduling

Min z

Subject to $\sum_{j=1}^{J(\bar{s})} \sum_{i=1}^{I(j)} a(k,\ell,\bar{s},i,j)\, x(\ell,\bar{s},i,j) + \tilde{u}_k(\ell,\bar{s}) \le z, \quad 1 \le k \le m$

$$\sum_{i=1}^{I(j)} x(\ell,\bar{s},i,j) = 1 \quad , \qquad 1 \le j \le J(\bar{s}) \qquad (\bar{P}_{\ell\bar{s}})$$

$$x(\ell,\bar{s},i,j) = 0 \text{ or } 1, \qquad 1 \le i \le J(j) \ , \quad 1 \le j \le J(\bar{s})$$

where

$$\tilde{u}_k(\ell,\bar{s}) = \sum_{\substack{s=1 \\ s \ne \bar{s}}}^{n} \sum_{\ell=1}^{L(s)} u_k(\ell,s)\, y(\ell,s)$$

and $\tilde{u}(\ell,\bar{s}) = [\tilde{u}_k(\ell,\bar{s})] \in E^m$ is called the <u>residual global utilization vector</u>.

This problem for school 1 in Figure 3 is illustrated in Figure 4. The run combinations for each route are selected to reduce the maximal components in the global utilization vector of Figure 3. These components equal to 5 in Figure 3 are reduced to 4 in Figure 4 with the selection of run combination 1 for each route.

With these new utilization vectors, the problem (P) of selecting a schedule for each school is solved again. This process is continued until the selection of the school schedules cannot be modified to improve the solution. The solution strategy is summarized in Figure 5.

The heuristic algorithm described in [17] has been used to solve problems (P) and $(\bar{P}_{\ell\bar{s}})$. The approach is to initialize the algorithm with a feasible solution; i.e. in problem (P) for each $1 \le s \le n$, choose exactly one variable $y(\ell,s) = 1$, $1 \le \ell \le I(s)$. Each block of variables (with the same

	Combined running for																	Residual global utilization vector	Utilization vector
	Route #1									Route #2									
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	
7:50																			
5th period before						1		1	1						1		1	1	4	4
8:00																			
4th period before			1	1		1	1	1	1			1	1		1	1	1	1	3	3
8:10																			
3rd period before	1	1	1	1	1		1			1	1	1	1	1		1			2	4
8:20																			
2nd period before	1	1			1					1	1			1					0	2
8:30																			
1st period before																			
8:40 School starting time																			⋮	⋮
⋮																				
3:10 School ending time																				
1st period after																				
3:20																			
2nd period after	1		1			1				1		1			1				1	3
3:30																			
3rd period after	1	1	1	1		1		1		1	1	1	1		1		1		2	4
3:40																			
4th period after		1		1	1		1	1	1		1		1	1		1	1	1	3	3
3:50																			
5th period after					1		1		1					1		1		1	4	4
4:00																			

Figure 4

Run combination selection

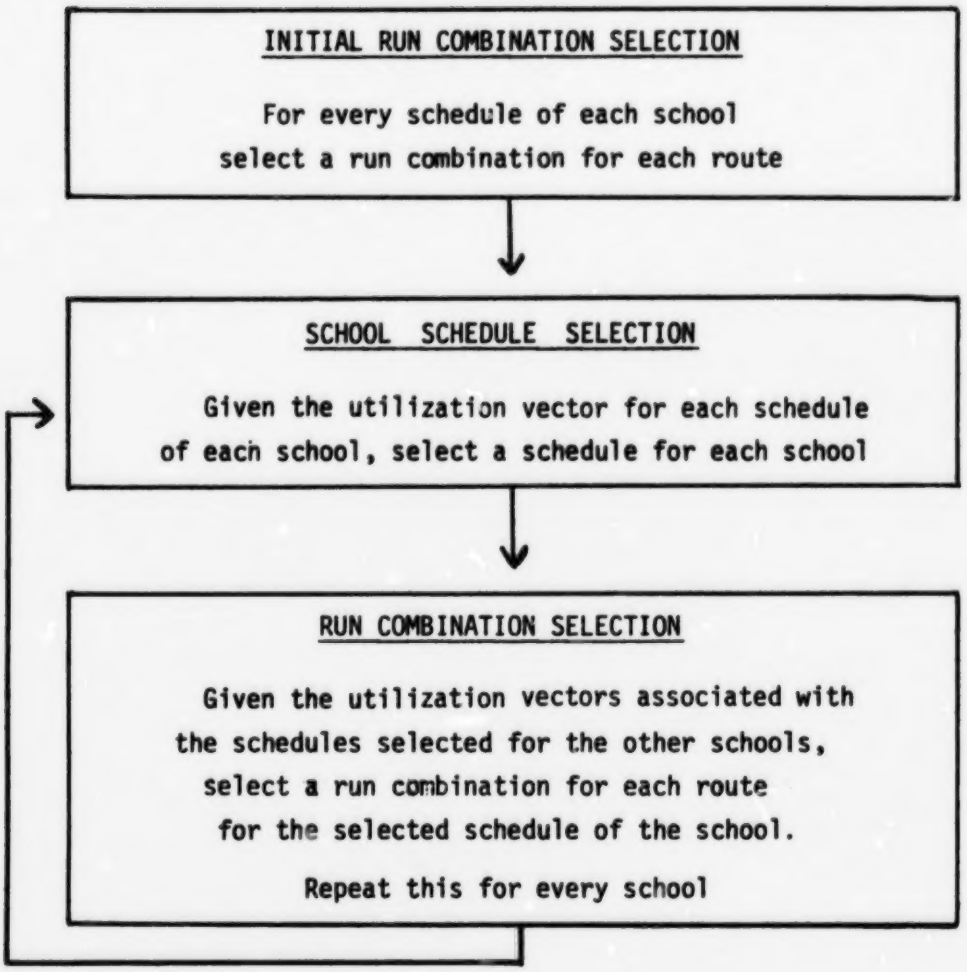
INITIAL RUN COMBINATION SELECTION
For every schedule of each school
select a run combination for each route
SCHOOL SCHEDULE SELECTION
Given the utilization vector for each schedule
of each school, select a schedule for each school
RUN COMBINATION SELECTION
Given the utilization vectors associated with
the schedules selected for the other schools,
select a run combination for each route
for the selected schedule of the school.
Repeat this for every school

Figure 5

SOLUTION STRATEGY

To minimize the number of routes executed simultaneously

index s) is analyzed cyclically. Assuming the values of the variables of the other blocks fixed, this block is scanned to determine the index $1 \leq \ell \leq I(s)$ for which the objective is minimized when $y(\ell,s) = 1$. When the solution cannot be improved, the algorithm is reinitialized with another initial feasible solution. The procedure is repeated with three different initial solutions and the best solution generated is retained. The value of this best solution is an approximation to a lower bound on the number of buses needed.

The size of problem (P) increases with the number of possible schedules for each school. In our tests reported in section 5, we were able to solve problems with $n = 73$ schools, $L(s) = 27$ possible schedules per school, a total of 273 routes, $I(j) = 9$ run combinations per route and $m = 35$ periods, (i.e. problems with 4428 variables and 35 constraints). Although the algorithm described in [17] is suboptimal, an error rate of less than 4% was observed in our numerical tests.

3.2 Improved approach

To improve the approach, we take into account the time linking constraint between routes executed successively by the same bus. The objective becomes a combination of minimizing the number of run combinations taking place simultaneously and of maximizing the number of routes that can be executed successively.

We proceed as follows. Consider some selection H of school schedules (starting and ending times). Suppose that for this selection, all routes are completed exactly ten minutes (one period) before school

starting times. For each pair of schools, p,q, determine $F_{am}(p,q)$ the number of routes associated with q that <u>follow</u> routes from p, that is, that can be executed after routes from p. Let n_s be the number of routes associated with school s,

$$F_{am}(s,\cdot) = \sum_q F_{am}(s,q), \quad \text{and} \quad F_{am}(\cdot,s) = \sum_p F_{am}(p,s) .$$

Then a lower bound $enb_{am}(H)$ on the estimated number of buses required is as follows :*

$$enb_{am}(H) = \text{Max} \{ \sum_s (n_s - F_{am}(s,\cdot))^+, \sum_s (n_s - F_{am}(\cdot,s))^+ \}$$

where $(n_s - F_{am}(s,\cdot))^+$ represents the minimum number of chains (buses) that terminate after a route of school s, while $(n_s - F_{am}(\cdot,s))^+$ represents the minimum number of chains that start at school s.

To illustrate, suppose we have the following matrix with entries $F_{am}(p,q)$ and associated graph (not always acyclic) for a problem with three schools p, q, r having 6, 8 and 11 routes respectively :

	p	q	r	$(n_s - F_{am}(s,\cdot))^+$
p	0	3	6	0
q	0	0	7	1
r	0	0	0	11
$(n_s - F_{am}(\cdot,s))^+$	6	5	0	

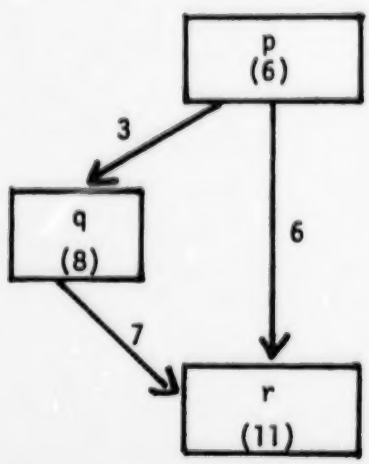

Figure 6 : <u>Availability matrix to minimize the number of buses</u>

* Strictly speaking $enb_{am}(H)$ is not a lower bound since we have restricted the problem by assuming that routes are completed in ten minutes before school starting times. However, in our tests, it has in fact always been a lower bound.

In our example above, $enb_{am}(H) = Max\{12,11\} = 12$.

Similarly, a lower bound can be obtained for the afternoon assuming that all routes are initiated exactly ten minutes after school ending times. This is denoted by $enb_{pm}(H)$. Then

$$enb(H) = Max\ \{enb_{am}(H),\ enb_{pm}(H)\}$$

is a lower bound on the number of buses for a selection H of school schedules.

Furthermore, for a selection H, we define B(H) to be

$$B(H) = \sum_{p} \sum_{q} (F_{am}(p,q) + F_{pm}(p,q)).$$

Let us denote by z(H) the value of z associated with the selection H of school schedules. The objective function of problem (P) is now modified according to both estimators z(H) and enb(H) :

$$Min\ [\alpha\ Max\{z(H),\ enb(H)\} + \beta\ Min\{z(H),\ enb(H)\} - \gamma\ B(H)].$$

We choose α, β and γ so that the algorithm sequentially minimizes the maximum of the two estimators, then minimizes the other one and finally maximizes the total number of feasible links between routes. In our tests, the values $\alpha = 1000\beta$ and $\gamma = 0.001\beta$ gave good results.

The original algorithm for solving problem (P) was described earlier. With the modified objective function the procedure is as follows. Each time z(H) is determined for some selection H, calculate enb(H). Then find Max{z(H), enb(H)} for all H that were considered. Suppose z*(H) is the

maximum value. Next, find the smallest enb(H) for all H with $z(H) = z^*$. Finally, for all H having $z(H) = z^*$ and enb(H) equal to the value found in the previous step, find Max B(H). The procedure is analogous if enb(H) is Max{z(H), enb(H)}. Note that the problem of selecting the run combination for each route as formulated in ($\overline{P}_{\ell s}$) is solved as before.

4. ROUTE SCHEDULING [10]

The preceding module for defining the school schedules also specifies the periods during which each route is executed. The purpose of this module is to specify the exact starting and ending time for each route. There is some flexibility due to the fact that a route taking 25 minutes can take place anywhere in the 30 minute period (or three periods) specified at the end of the preceding module. At this step, the notion of "period" is no longer necessary. The user can specify the exact schedule for each school and an interval for arrival or departure time of the routes. This may introduce more flexibility into the problem to be solved.

The problem is to form chains of routes to be executed successively with the objective of minimizing their total number. All the routes of a chain are executed by the same bus.

First, note that if the starting times were fixed for each route, then an acyclic oriented graph can be associated with the problem and an optimal solution can be determined using a variant of the maximal flow algorithm via Dilworth's theorem for partially ordered sets. However, in our case, the starting times are not known, but rather, must fit inside given time windows.

The approach used is heuristic and involves solving a series of transportation problems. The approach is described in [10]. An optimal approach for this problem is also described in [13,30].

5. RESULTS

After some preliminary testing of each module separately, the system was tested on data from the Saint-François school board in Drummondville (Québec). This section summarizes the most important aspects of this experiment.

In this region, there are 17,329 students, 22 urban schools (9 at high school level and 13 at elementary level), 25 rural schools (1 at high school level and 24 at elementary and kindergarten levels), and 8 connection points (related to the 9 urban high schools). The whole territory includes about 3,600 km^2 of urban development and 3,240 km^2 of rural area. Table 1 specifies the numbers of students transported in the urban and rural areas.

	Number of students
Urban area	2,772
Rural area :	
transported directly to school	4,472
transported to connection points	2,854
transported from connection point to school	2,854
Total	12,952

Table 1 : Students transported

The test was conducted with routes (or buses)with a capacity of 60 students. The constraints are specified in Table 2.

Upper bound constraint	Urban area	Rural area
Number of students per route	60	60
Length of route (high schools)	--	50 km
Lenght of route (elementary schools)	--	(40-45) km

Table 2 : The constraints

The number of routes generated are given in Table 3 indicating that the constraint on the number of students is the most binding in urban areas while the constraint on the length of the route is the most binding in rural areas (as expected).

Route type	Number of routes	Average number of students per route	Average length of route
Urban area	52	53	6,8 km
Rural area:			
to school	102	42	24.4 km
to connection point	63	48	33.0 km
from connection point to school	56	51	6.6 km
Total	273	47	19.4 km

Table 3 : Routes generated

Six different scenarios were studied to examine the sensitivity of the number of chains of routes (or the number of buses required) relative to the flexibility allowed on school schedules. In all these scenarios, the 9 high schools in the urban areas have a fixed schedule starting at 8:40 in the morning and a one hour lunch break. The interaction between their schedules becomes very complex because each of these schools receives students from several connection points. The different scenarios vary in the amount of flexibility present in scheduling the other schools.

Scenario I : All the other schools have a fixed schedule starting at 8:40 in the morning and a one hour lunch break.

Scenario II : All elementary schools form one group having the same schedule which is one of the six possible given that :

- starting time interval : 8:30 or 8:40 or 8:50
- lunch time length : 60 or 70 minutes.

The same flexibility is allowed in the schedule of the rural high school and the kindergartens.

Scenario III : The set of elementary schools is partitioned into 30 groups and the set of kindergartens into 11 groups. All the schools of a group have the same schedule, and each group has its own schedule. The flexibility of the schedule for each group and for the rural high school is specified as in scenario II.

Scenario IV : Similar to scenario II but with more flexibility. All elementary schools form one group having the same schedule which is one of the 21 possible given that :

- starting time interval : from 8:10 to 9:10
- lunch time length : 50 to 70 minutes.

Scenario V : Similar to scenario III with the flexibility specified as in scenario IV.

Scenario VI : The routes generated by our system have been scheduled according to the actual 1980-1981 schedules used by the school board. There are 29 groups with fixed schedules spreading from 8:00 to 9:20 for starting time and 50 to 90 minutes for lunch break. This scenario provides benchmark results for evaluating the system.

The results are summarized in Table 4. It was assumed that the speed for empty buses was 35 km/h in ruban areas and 65 km/h in rural areas. The time duration of productive routes was estimated using a chart provided by the Ministry of Transport. As a result, the average speed is 35.3 km/h. The final columns indicate the C.P.U. times required by the two modules. The results indicate that the number of chains decreases as the flexibility increases, as expected.

The results obtained show the importance of choosing a good schedule for each school in order to minimize the cost of the transportation of children. The worst schedule (all schools starting at the same time) requires 193 buses while the best schedule found requires only 126 buses to transport the same children. Even with the same school schedule, the

system saves 11 buses over the present quota, which would be a saving of over $200,000. Similar results and savings (5% - 10%) were also obtained for a smaller school board near Montréal.

Furthermore this reduction in the number of chains (or buses required) is not achieved at the expense of students' comfort since their average waiting time remains the same as flexibility increases. Finally, the increase in C.P.U. time required when flexibility increases is by far obviated by the gain in the reduction of the number of buses.

Figure 7 analyzes the arrival and departure time constraints. In all scenarios, it was required that each route be completed at least five minutes (eight for secondary) before school starting time in the morning and that each route be started at least five minutes (eight for secondary) after school ending time. A sensitivity analysis is performed using a *violation slack*. This is the total amount, summed over all routes in a chain, by which the time constraint may be violated. Figure 7 shows for each scenario, how the number of chains or buses varies as the violation slack is increased.

To conclude this section, Table 5 summarizes the computer times required by each module.

6. CONCLUSION

This modular school busing system performs well and generates good results in the opinion of school board authorities. The routes generated are very similar to those generated by these experienced people. For

Scenario	MORNING		AFTERNOON		C.P.U. TIME (sec.) REQUIRED		
	Number of chains	Average waiting time (min.)	Number of chains	Average waiting time (min.)	School scheduling	Route scheduling	Total
I	193	10,7	172 *	10.6	38	198	236
II	159	11.8	166	11.0	148	251	399
III	158	11.7	132	12.6	496	271	767
IV	145	10.2	102	9.5	279	562	841
V	126	11.2	116	10.9	3617	460	4077
VI	131	12.3	125	12.1	59	349	408

Table 4 : Comparison of school scheduling scenarios
(number of buses actually used for this problem was 142)

* The number of chains (buses) in the afternoon is generally smaller than in the morning. This follows from the fact that the school day length is ten minutes shorter for elementary schools than for high schools. Thus there is more flexibility in the afternoon.

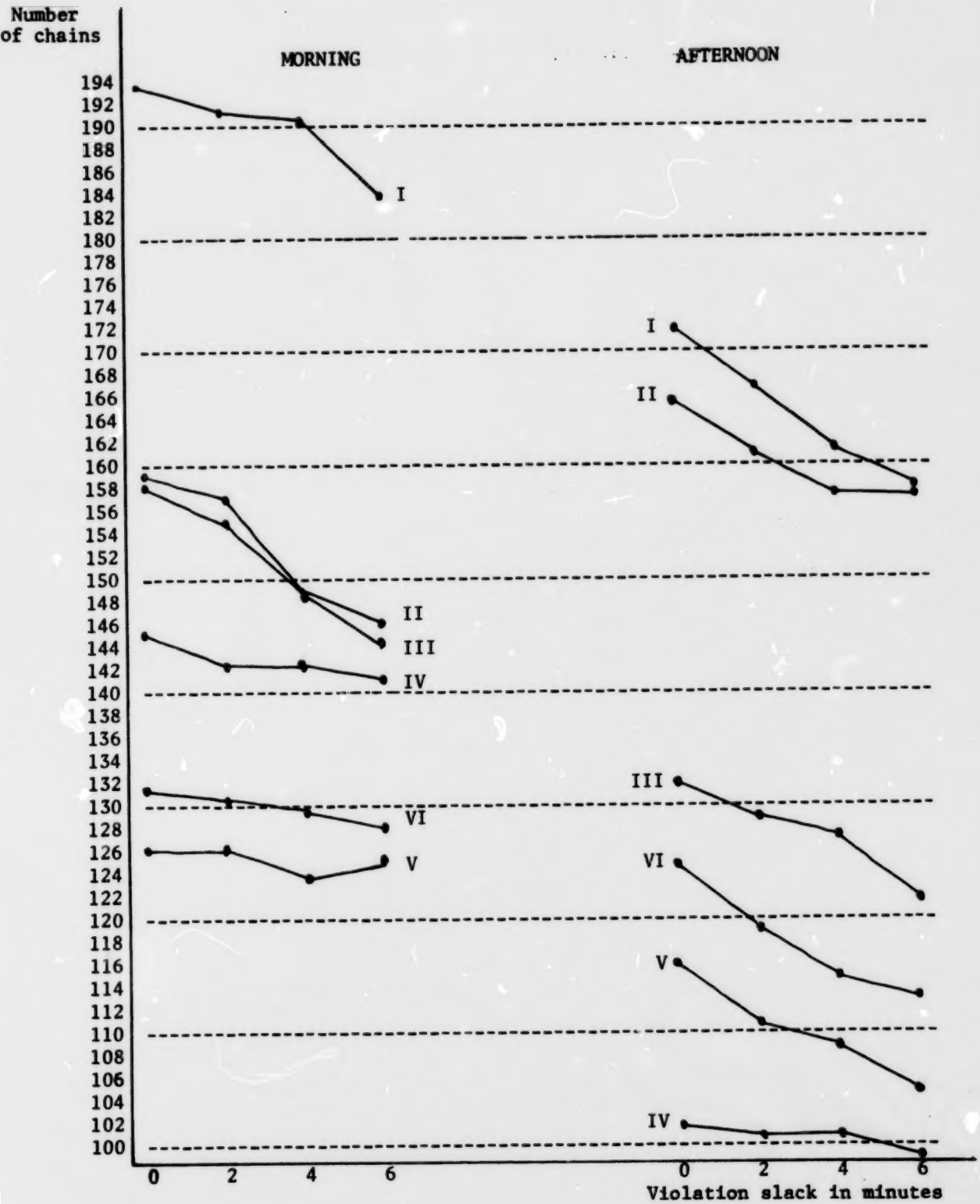

Figure 7

Sensitivity analysis of the time constraints on bus routes

Module		C.P.U. (sec.)	I.Ø (sec.)
Location		184	100
Urban route generation		894	575
Rural route generation		1459	580
Connection points to schools route generation		70	120
School and route scheduling:			
	Scenario I	236	275
	Scenario II	399	275
	Scenario III	767	275
	Scenario IV	841	275
	Scenario V	4077	275
	Scenario VI	408	275
Total:	Scenario I	2948	1650
	Scenario II	3111	1650
	Scenario III	3479	1650
	Scenario IV	6789	1650
	Scenario V	3120	1650
	Scenario VI		1650

Table 5

Computer times using the Cyber 173
at the Université de Montréal

example, the long routes (greater than one hour in duration) obtained by us and the school board were quite similar in number and in length.

The total computer time required by the system may seem large. But the size and the complexity of the problem justify this. Note that a team of two experienced persons currently takes about two months to do the job of planning student transportation. Compared to this, this system is inexpensive. In addition, the savings generated in the number of buses necessary more than justify the computer cost.

TRANSCOL has satisfied SIMEQ, the organization responsible for computer science applications in the field of education in the Province of Québec. It has been used by several school boards and plans are under preparation to implement TRANSCOL for all school boards desiring to use it.

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